

Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

EVENING

[Total No. of Questions: 09]

04 JUL 2022

[Total No. of Pages: 02]

Uni. Roll No.

Program: B.Tech. (Batch 2018 onward)

Semester: 4th

Name of Subject: Mathematics-111

Subject Code: BSCE- 101

Paper ID: 16180

Scientific calculator is Not Allowed

Detail of allowed codes/charts/tables etc. *N.A.*

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

- 1) Parts A and B are compulsory
- 2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
- 3) Any missing data may be assumed appropriately

Part – A

[Marks: 02 each]

Q1.

- a) State and prove first shifting property of Laplace Transform.
- b) Write the formula for Fourier integral of $f(x)$.
- c) Define order of an element of a group.
- d) State Modulation theorem of Fourier Transform.
- e) Prove that the proposition $p \vee \sim q$ is a tautology.
- f) Prove that in a distributive Lattice, if an element has a complement then this complement is unique.

Part – B

[Marks: 04 each]

Q2. State and prove Convolution theorem for Inverse Laplace Transform.

Q3. Evaluate the following:

(i) $L^{-1}\left(\frac{s+1}{s^2+s+1}\right)$ (ii) $L(\sqrt{t}e^{3t})$

Q4. Find Fourier Transform of $f(x) = \begin{cases} x & |x| \leq a \\ 0 & |x| > a \end{cases}$.

Q5. Use truth table to show that $p \Rightarrow q$ is logically equivalent to $\sim p \vee q$.

04 JUL 2022

- Q6. A Relation R on the set Z of all integers as follows:
 $m R n$ if and only if $(m + n)$ is even for all $m, n \in Z$. Is R a partial order relation?
 Justify your answer with a suitable example.
- Q7. Prove that the set $\{0,1,2,3,4\}$ is a finite abelian group under addition modulo 5.

Part – C

[Marks: 12 each]

- Q8. (a) Prove that a subgroup H of a group G is normal if and only if $g^{-1}hg \in H$, for all $h \in H, g \in G$. (6)
- (b) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. (6)

OR

- (a) Consider the POSET $A = (\{1,2,3,4,6,9,12,18,36\}, /)$. Find the greatest lower bound and the least upper bound of the sets $\{6,18\}$ and $\{4, 6, 9\}$. (6)
- (b) Define a Lattice with a suitable example and Prove that the product of two Lattices is a Lattice. (6)
- Q9. Use Fourier cosine transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, x > 0, t > 0$ subject to the conditions $u_x(0, t) = 0, u(x, t)$ is bounded and $u(x, 0) = \begin{cases} x; & 0 \leq x \leq 1 \\ 0; & x > 1 \end{cases}$.

OR

Solve the following differential equation using Laplace transform:

$$2 \frac{d^2 y}{dt^2} - \frac{dy}{dt} - y = \cos t, y(0) = 1, y'(0) = 0.$$
